Answer on Question #45231 – Math – Matrix - Tensor Analysis

1) M = [[-2,1,0], [-11,4,1],[-18,6,1]]

a) solve the equation M [[x] , [y], [z]] = 3 [[x] , [y] , [z]]

b) solve the equation M [[x], [y], [z]] = 4 [[x] , [y], [z]]

Solution

a)

$$\begin{cases} (-2-3)x + y + 0\cdot z = 0 \\ -11x + (4-3)y + 1\cdot z = 0 \\ -18x + 6y + (1-3)\cdot z = 0 \end{cases} \rightarrow \begin{cases} (-5)x + y = 0 \\ -11x + y + 1\cdot z = 0 \\ -18x + 6y + (-2)\cdot z = 0 \end{cases}$$

$$y = 5x$$

$$-11x + 5x + 1\cdot z = 0 \rightarrow z = 6x$$

$$-18x + 6\cdot 5x + (-2)\cdot 6x = 0 \rightarrow 0\cdot x = 0 \rightarrow x = t, t \in R.$$

So

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = t\begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix}.$$

b)

$$\begin{cases} (-2-4)x + y + 0\cdot z = 0 \\ -11x + (4-4)y + 1\cdot z = 0 \\ -18x + 6y + (1-4)\cdot z = 0 \end{cases} \rightarrow \begin{cases} (-6)x + y = 0 \\ -11x + 1\cdot z = 0 \\ -18x + 6y + (-3)\cdot z = 0 \end{cases}$$

$$y = 6x$$

$$z = 11x$$

$$-18x + 6\cdot 6x + (-3)\cdot 11x = 0 \rightarrow -15x = 0 \rightarrow x = 0.$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

2) A matrix S has eigenvalues 2, -3 and 5 with corresponding eigenvectors v1 = [[1] , [-3] , [-2]] , v2 = [[-2], [7], [5]] and v3 = [[0], [0], [1]] respectively.

a) write down the values of Sv1, Sv2 and Sv3.

b) evaluate S[[0], [1], [0]]

d) hence or otherwise find matrix S

Solution

a)

$$\mathrm{S}v_1 = 2v_1 = 2\begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \\ -4 \end{pmatrix}$$

$$\mathrm{Sv}_2 = -3v_2 = -3\begin{pmatrix}-2\\7\\5\end{pmatrix} = \begin{pmatrix}6\\-21\\-15\end{pmatrix}$$

$$\mathrm{Sv}_3 = 5v_3 = 5\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\5\end{pmatrix}$$

b)

$$\begin{pmatrix}0\\1\\0\end{pmatrix} = a\begin{pmatrix}1\\-3\\-2\end{pmatrix} + b\begin{pmatrix}-2\\7\\5\end{pmatrix} + c\begin{pmatrix}0\\0\\1\end{pmatrix} \to$$

$$2a = b;\ -3a + 7\cdot 2a = 1 \to a = \frac{1}{11}, b = \frac{2}{11};\ -2\cdot\frac{1}{11} + \frac{2}{11}\cdot 5 + c = 0 \to c = -\frac{4}{11}.$$

So

$$\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{1}{11}\begin{pmatrix}1\\-3\\-2\end{pmatrix} + \frac{2}{11}\begin{pmatrix}-2\\7\\5\end{pmatrix} - \frac{4}{11}\begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{1}{11}v_1 + \frac{2}{11}v_2 - \frac{4}{11}v_3.$$

$$S\begin{pmatrix}0\\1\\0\end{pmatrix} = S\left(\frac{1}{11}v_1 + \frac{2}{11}v_2 - \frac{4}{11}v_3\right) = \frac{1}{11}\mathrm{Sv}_1 + \frac{2}{11}\mathrm{Sv}_2 - \frac{4}{11}\mathrm{Sv}_3 = \frac{1}{11}\begin{pmatrix}2\\-6\\-4\end{pmatrix} + \frac{2}{11}\begin{pmatrix}6\\-21\\-15\end{pmatrix} - \frac{4}{11}\begin{pmatrix}0\\0\\5\end{pmatrix}.$$

$$S\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{1}{11}\begin{pmatrix}2+2\cdot 6-4\cdot 0\\-6+2\cdot(-21)-4\cdot 0\\-4+2\cdot(-15)-4\cdot 5\end{pmatrix} = \frac{1}{11}\begin{pmatrix}14\\-48\\-54\end{pmatrix}.$$

d) We need to find $S\begin{pmatrix}1\\0\\0\end{pmatrix}$.

$$\begin{pmatrix}1\\0\\0\end{pmatrix} = d\begin{pmatrix}1\\-3\\-2\end{pmatrix} + e\begin{pmatrix}-2\\7\\5\end{pmatrix} + f\begin{pmatrix}0\\0\\1\end{pmatrix} \to$$

$$7e = 3d \to e = \frac{3}{7}d; d - 2\cdot\frac{3}{7}d = 1 \to d = 7, e = 3;\ -2\cdot 7 + 5\cdot 3 + f = 0 \to f = -1.$$

$$\begin{pmatrix}1\\0\\0\end{pmatrix} = 7v_1 + 3v_2 - v_3.$$

$$S\begin{pmatrix}1\\0\\0\end{pmatrix} = S(7v_1 + 3v_2 - v_3) = 7Sv_1 + 3Sv_2 - Sv_3 = 7\begin{pmatrix}2\\-6\\-4\end{pmatrix} + 3\begin{pmatrix}6\\-21\\-15\end{pmatrix} - \begin{pmatrix}0\\0\\5\end{pmatrix} = \begin{pmatrix}32\\-105\\-78\end{pmatrix}.$$

The rows of S are transposed vectors $S\begin{pmatrix}1\\0\\0\end{pmatrix}$, $S\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $S\begin{pmatrix}0\\0\\1\end{pmatrix}$.

The matrix S is

$$S = \begin{pmatrix}32 & -105 & -78\\ \frac{14}{11} & -\frac{48}{11} & -\frac{54}{11}\\ 0 & 0 & 5\end{pmatrix}.$$