The common ratio of geometric progression is 2 and the sum of the first eight terms is 1020. Find the first term of the progression.

Solution.

The sum of n first terms of geometric progression is calculated as $(q \neq 1)$

$$S_n = a_1 + \cdots + a_n = a_1 \frac{1-q^n}{1-q}. \tag{1}$$

A constant q is called the common ratio.

In general, the terms of a geometric progression are expressed as

$$a_n = a_1 q^{n-1}, n = 1, 2, 3, \dots$$

By statement of the problem, $q = 2, S_8 = 1020$.

Substitute them for (1) and obtain

$$a_1 \frac{1-2^8}{1-2} = 1020, \quad a_1 \frac{2^8-1}{2-1} = 1020, \quad a_1(2^8 - 1) = 1020, \text{ hence } a_1 = \frac{1020}{2^8-1}, \quad a_1 = \frac{1020}{255}, \quad a_1 = 4.$$

Answer: the first term of the progression is 4.