Answer on Question #45193 – Math – Statistics and Probability

Question. A company manufactures 4000 cars in a year with a probability that a part will be defective is 0.002. Find the probability that company produces

a) 4 cars with defective

b) at least 4 cars

c) at most 4 cars with defective.

Solution. The Poisson's approximation of Binomial distribution: $P(n,k) \approx \frac{\lambda^k}{k!}e^{-\lambda}, \lambda = np$. In our case $n = 4000, p = 0.002, \lambda = 4000 \cdot 0.002 = 8$. Let ξ be the number of cars with defective.

a) $P(\xi = 4) \approx \frac{8^4}{4!}e^{-8} \approx 0.057.$

c) $P(\xi \leq 4) = P(\xi = 0) + P(\xi = 1) + P(\xi = 2) + P(\xi = 3) + P(\xi = 4) \approx \frac{8^0}{0!}e^{-8} + \frac{8^1}{1!}e^{-8} +$

$+\frac{8^2}{2!}e^{-8} + \frac{8^3}{3!}e^{-8} + \frac{8^4}{4!}e^{-8} = 297e^{-8} \approx 0.0996.$

b) $P(\xi \geq 4) = 1 - P(\xi < 4) = 1 - P(\xi = 0) - P(\xi = 1) - P(\xi = 2) - P(\xi = 3) \approx$

$\approx 1 - \frac{8^0}{0!}e^{-8} - \frac{8^1}{1!}e^{-8} - \frac{8^2}{2!}e^{-8} - \frac{8^3}{3!}e^{-8} \approx 1 - 0.042 = 0.958.$

Answer.

a) $P(\xi = 4) \approx 0.057$

b) $P(\xi \geq 4) \approx 0.958$

c) $P(\xi \leq 4) \approx 0.0996.$