Answer on Question #45190 – Math - Statistics and Probability

Problem.

The sales in a two wheeler showroom is exponentially distributed with mean equal to 4. If two days are selected at random, what is the probability that
a) on both days the sales is over 5 units **b)** the sale is over 5 units on at least one of the two days.

Solution.

The probability density function (pdf) of an exponential distribution with mean 4 is

$$f(x) = \begin{cases} \dfrac{e^{-\frac{x}{4}}}{4} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

The probability that on one day the sale is over 5 units equals

$$\int_5^{+\infty} f(x)\ dx = \int_5^{+\infty} \frac{e^{-\frac{x}{4}}}{4}\ dx = -\int_5^{+\infty} e^{-\frac{x}{4}}\ d\left(-\frac{x}{4}\right) = -e^{-\frac{x}{4}}\Big|_5^{+\infty} \approx 0.287.$$

The probability that on one day the sale is below 5 units equals

$$1 - 0.287 = 0.713.$$

a) The probability that on both days the sales is over 5 units equals

$$P(A \cap B) = 0.287^2 \approx 0.082.$$

b) The probability that the sale is over 5 units on at least one of the two days equals

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.287 + 0.287 - 0.287^2 = 1 - P(\bar{A} \cap \bar{A}) = 1 - 0.713^2 \approx 0.491.$$

Answer: a) 0.082, b) 0.491.