Answer on Question #45173 – Math - Differential Calculus | Equations

Under what conditions the solution of the first order ordinary differential equation exist?

Furthermore, without solving the following initial value problem determine the interval in which the solution is certain to exist.

dy/dx + (tan x) y = sin x: y (π/4) =0

Solution.

- **Consider the first order $y' = f(x,y)$ with initial condition $y(x_0) = y_0$**

where $f(x,y)$ is bounded in the neighborhood of the initial point, i.e.,

$|f(x,y)\} \leq M < \infty \;\; in \;\; R = \{|x - x_0| \leq a, |y - y_0| \leq b\}$

If $f(x,y)$ is continuous in the neighborhood region $h = min\{a, \frac{b}{M}\}$, the solution of this initial value problem in the region h exists.

- **Now, consider the following IVP:**

$y' + p(x)y = g(x), \qquad y(x_0) = y_0$

As we know, if $p(x)$ and $g(x)$ are continuous functions on an open interval and the interval contains x_0, then there is a unique solution to the IVP on that interval.

In our case $p(x) = tanx, \; g(x) = sinx, \; x_0 = \frac{\pi}{4}, \; y_0 = 0.$

$sinx$ is continuous on $(-\infty, \infty)$,

$tanx$ is continuous on interval $(0, \frac{\pi}{2})$, which contains $\frac{\pi}{4}$.

Therefore on interval $(0, \frac{\pi}{2})$ the solution certainly exists.