Answer on Question #45126 – Math – Multivariable Calculus

1. Evaluate the integral of $f(x, y, z) = x + 2y - z$ over the cylinder bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 1$.

Solution.

To evaluate the integral, we will use cylindrical coordinates:

$x = \rho cos\varphi$,

$y = \rho sin\varphi$,

$z = z$.

Then we have:

$\int_0^R d\rho \left(\rho \int_0^{2\pi} d\varphi \int_0^1 f(\rho cos\varphi, \rho sin\varphi, z)\, dz\right)$.

From $x^2 + y^2 = R^2 = 4$ we have that $R = 2$.

Hence, we can evaluate our integral:

$$\int_0^2 d\rho \left(\rho \int_0^{2\pi} d\varphi \int_0^1 dz(\rho cos\varphi + 2\rho sin\varphi - z)\right) = \int_0^2 \rho^2 d\rho \int_0^{2\pi} os\varphi d\varphi \int_0^1 dz +$$
$$+2 \int_0^2 \rho^2 d\rho \int_0^{2\pi} sin\varphi d\varphi \int_0^1 dz - \int_0^2 \rho d\rho \int_0^{2\pi} d\varphi \int_0^1 zdz = \frac{\rho^3}{3}|_0^2 sin\varphi|_0^{2\pi} z|_0^1 +$$
$$2\frac{\rho^3}{3}|_0^2 (-cos\varphi)|_0^{2\pi} z|_0^1 - \frac{\rho^2}{2}|_0^2 \varphi|_0^{2\pi} \frac{z^2}{2}|_0^1 = \frac{8}{3} \cdot 0 \cdot 1 + 2 \cdot \frac{8}{3} \cdot 0 \cdot 1 - \frac{4}{2} \cdot 2\pi \cdot \frac{1}{2} = -2\pi.$$

Answer:

hence, our integral is -2π.