Answer on Question #45125 – Math - Algebra

Using the given zero, find one other zero of $f(x)$. Explain the process you used to find your solution.

$1-2i$ is a zero of $f(x)=x^4-2x^3+6x^2-2x+5$.

Solution.

If function $f(x)$ has a zero x then following equation holds true:

$$x^4-2x^3+6x^2-2x+5=0$$

If equation with real coefficients has a root $x=1-2i$, then equation also has the root $x=1+2i$ (complex conjugate roots). Next, divide the polynomial $x^4-2x^3+6x^2-2x+5$ by a factor:

$$(x-1+2i)(x-1-2i)=(x-1)^2-(2i)^2=x^2-2x+5$$

We want to divide one polynomial by another, using "long division":

$$\begin{array}{l|l} x^4-2x^3+6x^2-2x+5 & \underline{x^2-2x+5} \\ \underline{x^4-2x^3+5x^2} & x^2+1 \\ \qquad\quad \underline{x^2-2x+5} & \\ \qquad\qquad\qquad 0 & \end{array}$$

Then our equation takes the form:

$$(x^2-2x+5)(x^2+1)=0$$

Equating to zero each multiple in the left-hand side of the last equation , we obtain the new equation:

$$x^2+1=0$$
$$x=\pm\sqrt{-1}$$
$$x=\pm i$$

We already know that equation $x^2-2x+5=0$ has roots $x=1-2i$ and $x=1+2i$

Answer: we found all the zeros of the function $f(x)=x^4-2x^3+6x^2-2x+5$, using the given zero $x=1-2i$.

All zeros:

$$x=1-2i$$

$x = 1 + 2i$
$x = i$
$x = -i$