Answer on Question #45124 – Math - Algebra

Question:

State how many imaginary and real zeros the function has.

$f(x) = x^3 - 20x^2 + 123x - 216$

Answer:

Using Descartes' Rule of Signs we get 3 sign changes, so 3 real or 1 real and 2 imaginaries.

The general cubic equation has the form

$$ax^3 + bx^2 + cx + d = 0$$

Every cubic equation has at least one solution x. We can distinguish several possible cases using the discriminant

$$\Delta = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2$$

If $\Delta > 0$, then the equation has three distinct real roots.

If $\Delta = 0$, then the equation has a multiple root and all its roots are real.

If $\Delta < 0$, then the equation has one real root and two complex conjugate roots.

In our case discriminant equals:

$$\Delta = 18 \cdot (-20) \cdot 123 \cdot (-216) - 4 \cdot (-20)^3 \cdot (-216) + (-20)^2 \cdot 123^2 - 4 \cdot 123^3 - 27 \cdot (-216)^2 = 900 > 0$$

Answer: 3 real, 0 imaginaries.