Find the vertices and foci of the hyperbola with equation quantity x minus three squared divided by sixteen minus the quantity of y plus four squared divided by nine = 1

Solution:

Equation in standard form for the hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
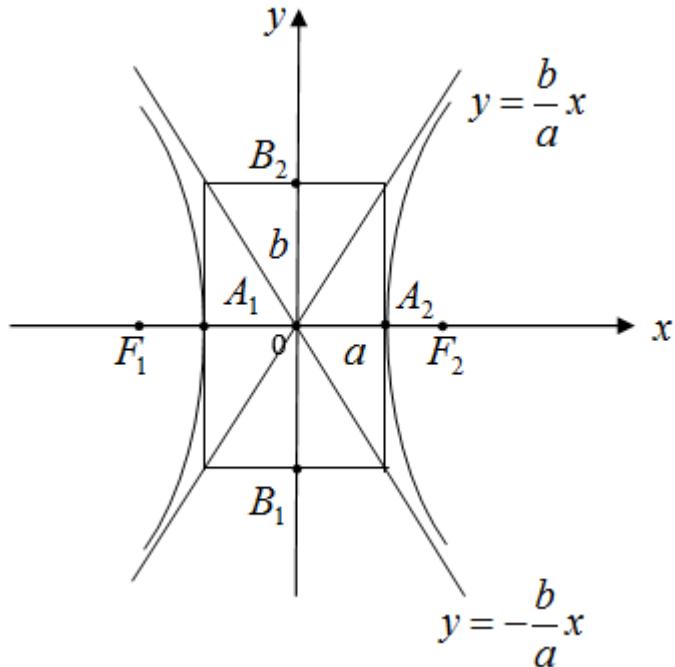
Where F_1 and F_2 are at $(0, -c)$ and $(c, 0)$ – foci of the hyperbola.

Our equation:

$$\frac{(x-3)^2}{16} - \frac{(y+4)^2}{9} = 1$$

From the equation, clearly the center is at $(h, k) = (3, -4)$.

$$a^2 = 16; \ a = 4$$

$$b^2 = 9; b = 3$$

Since the vertices $a = 4$ units to either side, then they are at $(7, -4)$ and $(-1, -4)$. The equation $c^2 = a^2 + b^2$ gives me $c^2 = 16 + 9 = 25$, so $c = 5$, and the foci, being 5 units to either side of the center, must be at $(8, -4)$ and $(-2, -4)$.

Answer: vertices $(7, -4)$ and $(-1, -4)$; foci $(8, -4)$ and $(-2, -4)$.