Answer on Question #45108 – Math - Calculus

Give an example of a function with both a removable and a non-removable discontinuity.

Solution:

Definition. Let *f* be a function and let a be a point in its domain. Then *f* is continuous at the single point *x = a* provided

$$\lim_{x\to a} f(x) = f(a)$$

If f is continuous at each point in its domain, then we say that f is continuous.

We are concerned now with determining continuity at the point *x = a* for a piecewise-defined function. If a piecewise-defined function f is not continuous at x = a , then there is a discontinuity which can take one of the following forms:

(i) $\lim_{x\to a} f(x)$ exists, but *f (a)* is either not defined or does not equal the limit. Then there is a “hole in the graph,” which is formally called a ***removable discontinuity***.

(ii) If the one-sided limits are finite but not equal,

$$\lim_{x\to -a} f(x) \neq \lim_{x\to +a} f(x)$$

then there is a jump discontinuity, which is also called a ***non-removable discontinuity***.

(iii) If one or both of the one-sided limits is infinite, then there is a vertical asymptote, which is called an infinite discontinuity.

Let us consider piecewise-defined function (an example of a function with both a removable and a non-removable discontinuity)

$$f(x) = \begin{cases} x, & if\ x < 0, \\ 1, & if\ x = 0 \\ x^2, & if\ 0 < x < 1, \\ x+1, & if\ x \geq 1, \end{cases}$$

If a piecewise-defined function f is not continuous at ***x = 0** and **x=1*** then there are a discontinuity. For *x = 0* we have

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0-} f(x) = 0 \neq f(0) = 1$$

Limit of f (x) exists, but f (0) is not equal to the limit. Then there is a “hole in the graph” or **removable discontinuity.** For *x = 1* we have

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} x^2 = 1 \neq \lim_{x\to +1} f(x) = \lim_{x\to +1} (x+1) = 2$$

then there is a jump discontinuity, which is also called **a non-removable discontinuity.**