Answer on Question #45103 – Math – Analytical Geometry

Find the center, vertices, and foci of the ellipse with equation 3x2 + 6y2 = 18

Solution:

To be able to read any information from this equation, we'll need to rearrange it to get "=1", so I'll divide through by 18.

This gives

$$3x^2 + 6y^2 = 18$$
$$\frac{3x^2}{18} + \frac{6y^2}{18} = \frac{18}{18}$$
$$\frac{x^2}{6} + \frac{y^2}{3} = 1$$

Since $x^2 = (x - 0)^2$ and $y^2 = (y - 0)^2$, the equation above is really:

$$\frac{(x-0)^2}{6} + \frac{(y-0)^2}{3} = 1$$

Then the center is at $(h, k) = (0, 0)$. I know that the a^2 is always the larger denominator (and b^2 is the smaller denominator), and this larger denominator is under the variable that parallels the longer direction of the ellipse. Since 6 is larger than 3, then $a^2 = 6$, $a = \pm\sqrt{6}$, and this ellipse is wider (paralleling the x-axis) than it is tall. The value of a also tells me that the vertices are $\sqrt{6}$ units to either side of the center, at ($-\sqrt{6}$, 0) and ($\sqrt{6}$, 0).

Let's find co-vertices of the ellipse:

$$b^2 = 3$$
$$b = \pm\sqrt{3}$$

Co-vertices: ($-\sqrt{3}$, 0) and ($\sqrt{3}$, 0).

To find the foci, we need to find the value of c. From the equation, I already have a^2 and b^2, so:

$$a^2 - c^2 = b^2$$
$$6 - c^2 = 3$$
$$c^2 = 3$$
$$c = \sqrt{3}$$

Then the value of c is 3, and the foci are three units to either side of the center, at ($-\sqrt{3}$, 0) and ($\sqrt{3}$, 0)

Answer: center (0,0),

vertices: ($-\sqrt{6}$, 0) and ($\sqrt{6}$, 0), ($-\sqrt{3}$, 0) and ($\sqrt{3}$, 0).

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