Question:

Find the new equation of the conicoid $9x^2 + 16y^2 - 36z^2 - 36x - 72z = 144$ when the coordinate system is changed into a new system with the same origin at (-2, 0, 1) and direction ratios same as the old system.

Solution.

Let denote axes of new coordinate system u,v,w.

As a new system is with the origin at (-2, 0, 1) and direction ratios same as the old system, hence we can conclude that u=x-2, v=y, w=z+1.

Hence, we have

Substituting this into the equation of the conicoid in the old system, we get

$$9(u+2)^{2} + 16v^{2} - 36(w-1)^{2} - 36(u+2) - 72(w-1) = 144$$

After simplification, we get

$$9u^2 + 16v^2 - 36w^2 - 144 = 0$$

So, the the new equation of the conicoid $9x^2 + 16y^2 - 36z^2 - 36x - 72z = 144$ is

$$9u^2 + 16v^2 - 36w^2 - 144 = 0$$

Answer. $9u^2 + 16v^2 - 36w^2 - 144 = 0.$