Answer on Question #45097 – Math - Analytic Geometry

Problem.

At what point the origin must be shifted so that linear terms in the conicoid x^2 + 2y^2 - z^2 - 2yz + 2xz + x - 3y + z + 4 = 0 vanish? Justify.

Solution.

Suppose that the origin is shifted to point (a, b, c), (x, y, z) are coordinates in old system, (x', y', z') are coordinates in new system. Then $x = x' + a$, $y = y' + b$, $z = z' + c$. After substation $x = x' + a$, $y = y' + b$, $z = z' + c$ into $x^2 + 2y^2 - z^2 - 2yz + 2xz + x - 3y + z + 4 = 0$ we will obtain

$$(x')^2 + 2x'a + a^2 + 2(y')^2 + 4y'b + 2b^2 - (z')^2 - 2z'c - c^2 - 2y'z' - 2y'c - 2z'b - 2bc + 2x'z' + 2x'c + 2z'a + 2ac + x' + a - 3y' - 3b + z' + c + 4 = 0$$

or

$$(x')^2 + 2(y')^2 - (z')^2 - 2y'z' + 2x'z' + x'(2a + 2c + 1) + y'(4b - 2c - 3) - z'(2c + 2b - 2a - 1) + a^2 + 2b^2 - c^2 - 2bc + 2ac + a - 3b + c + 4 = 0.$$

The linear terms should vanish, so

$$\begin{cases} 2a + 2c + 1 = 0; \\ 4b - 2c - 3 = 0; \\ 2c + 2b - 2a - 1 = 0; \end{cases}$$

$$\begin{cases} 2a + 2c + 1 = 0; \\ 4b - 2c - 3 = 0; \\ (2c + 2b - 2a - 1) + (2a + 2c + 1) = 2b + 4c = 0. \end{cases}$$

$$\begin{cases} 2a + 2c + 1 = 0; \\ 4b - 2c - 3 = 0; \\ b = -2c. \end{cases}$$

$$\begin{cases} a = -0.2; \\ c = -0.3; \\ b = 0.6. \end{cases}$$

Answer: $(-0.2, -0.3, 0.6)$.