Problem.

Using projection show that the line passing through (-1, 8, 8) and (6, 2, 0) is perpendicular to the line passing through (4, 2, 3) and (2, 1, 2).

Solution.

The direction vector of the first line is $\vec{a}(7, -6, -8)$ and the direction vector of the second line is $\vec{b}(-2, -1, -1)$. The projection of the vector \vec{a} on the vector \vec{b} is equal to

$$\overrightarrow{a_0} = \frac{(\overrightarrow{a}, \overrightarrow{b})}{\left|\overrightarrow{b}\right|^2} \overrightarrow{b},$$

where (\vec{a}, \vec{b}) is inner product of (\vec{a}, \vec{b}) .

$$\overrightarrow{a_0} = \frac{7 \cdot (-2) + (-6) \cdot (-1) + (-8) \cdot (-1)}{(-2)^2 + (-1)^2 + (-1)^2} \overrightarrow{b} = \overrightarrow{0},$$

so the projection of the vector \vec{a} on the vector \vec{b} is equal to $\vec{0}$. Hence the vector \vec{a} is perpendicular to the vector \vec{b} or the line passing through (-1, 8, 8) and (6, 2, 0) is perpendicular to the line passing through (4, 2, 3) and (2, 1, 2) (\vec{a} and \vec{b} are direction vectors of this lines).