Problem.

Let R be the point which divides the line segment joining P(2,1,0) and Q(-1,3,4) in the ratio 1:2 such that PR < PQ.

Find the equation of the line passing through R and parallel to the line x/2=y/1=z/3 .

Solution.

Suppose that R has coordinates (a, b, c). Then $\overrightarrow{PQ}$ has coordinates $(-3,2,4)$ and $\overrightarrow{PR}$ has coordinates $(a - 2, b - 1, c)$. $\overrightarrow{PR} = \frac{1}{3}\overrightarrow{PQ}$, so $\frac{1}{3}(-3,2,4) = (a - 2, b - 1, c)$. Therefore $a = 1, b = \frac{5}{3}, c = \frac{4}{3}$.

The line parallel to $\frac{x}{2} = \frac{y}{1} = \frac{z}{3}$ has direction vector $(2,1,3)$. Hence the equation of the line passing through R and parallel to the line $\frac{x}{2} = \frac{y}{1} = \frac{z}{3}$ is $\frac{x-1}{2} = \frac{y-\frac{5}{3}}{1} = \frac{z-\frac{4}{3}}{3}$, i.e. $\frac{x-1}{2} = \frac{3y-5}{3} = \frac{3z-4}{9}$.

Answer: $\frac{x-1}{2} = \frac{3y-5}{3} = \frac{3z-4}{9}$.