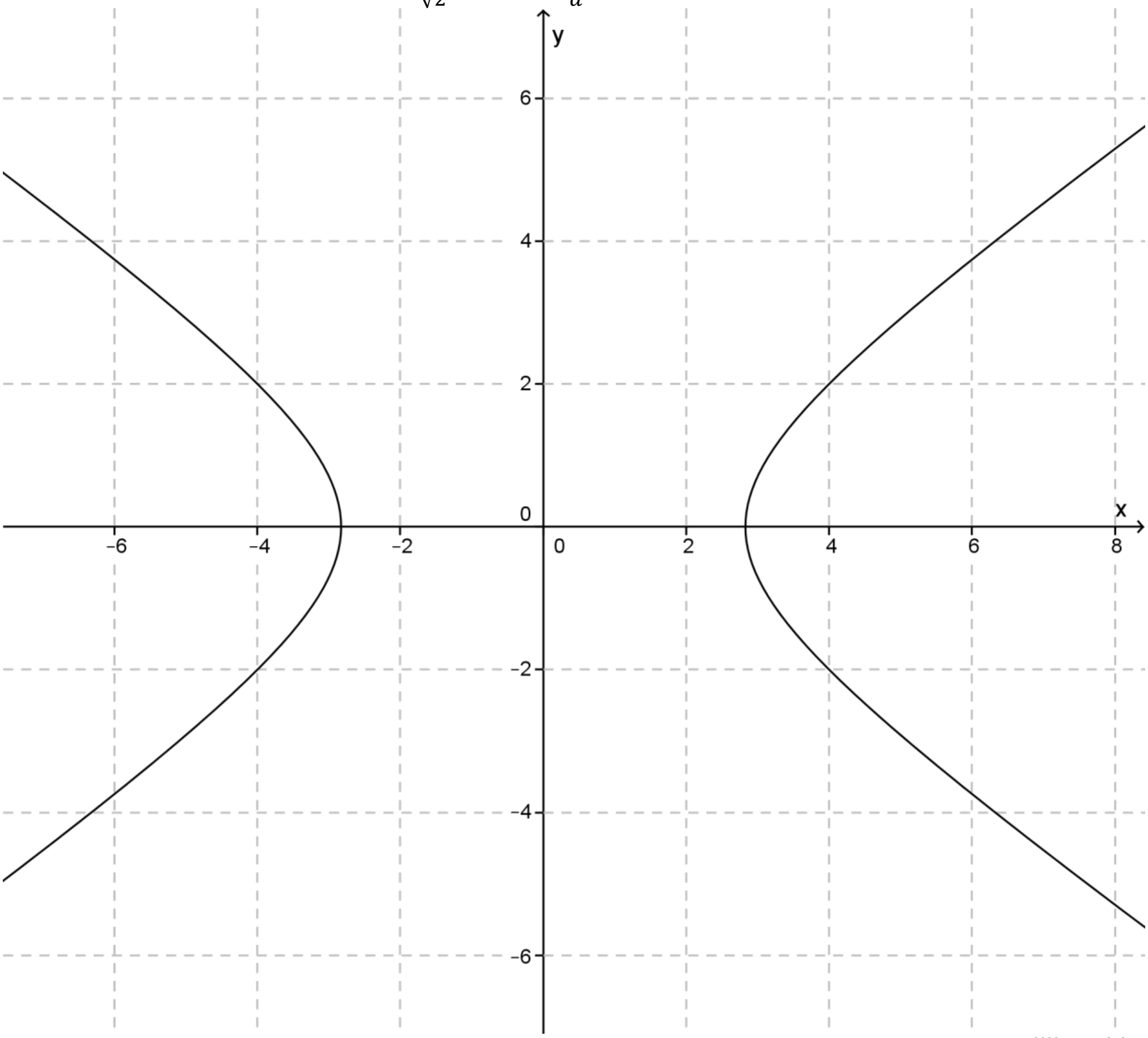
Answer on Question #45089 – Math - Analytic Geometry

Problem.

Find the vertices, eccentricity, foci and asymptotes of the hyperbola x^2/8-y^2/4=1.
Also trace it.
Under what conditions on (lamda) the line x - (lamda)y +2 = 0 will be tangent to this hyperbola? Explain geometrically.

Solution.

The equation of the hyperbola is $\frac{x^2}{8}-\frac{y^2}{4}=1$, so $a^2=8$ and $b^2=4$ or $a=2\sqrt{2}$ and $b=2$. Then $c^2=a^2+b^3=8+4=12$, so $c=2\sqrt{3}$, the vertices are $(-2\sqrt{2},0)$ and $(2\sqrt{2},0)$ $((-a,0)$ and $(a,0))$, the eccentricity is $e=\frac{c}{a}=\frac{2\sqrt{3}}{2\sqrt{2}}=\frac{\sqrt{6}}{2}$, the foci are $(-2\sqrt{3},0)$ and $(2\sqrt{3},0)$ $((-c,0)$ and $(c,0))$, the asymptotes are $y=\pm\frac{1}{\sqrt{2}}x$ $(y=\pm\frac{b}{a}x)$.



The equation of tangent line to this hyperbola that passes throught point (x_0,y_0) is $\frac{xx_0}{8}-\frac{yy_0}{4}=1$.
We should find such λ that $-\frac{x}{2}+\frac{\lambda y}{2}=1$ $(x-\lambda y+2=0)$.
Then

$$-\frac{1}{2}=\frac{x_0}{8} \text{ and } -\frac{\lambda}{2}=\frac{y_0}{4}$$

or

$$x_0=-4 \text{ and } y_0=-2\lambda,$$

but $\frac{x_0^2}{8}-\frac{y_0^2}{4}=1$, so $2-\lambda^2=1$. Therefore $\lambda=\pm1$.

Hence there are two tangent lines $x + y + 2 = 0$ (blue line) and $x - y + 2 = 0$ (red line). There are two tangent lines because the hyperbola is symmetric with respect to x-axis.