

Answer on Question #45080 – Math - Analytic Geometry

Problem

Find the reciprocal cone of the cone $x^2+z^2-2yz+4zx=0$.

Solution

The reciprocal cone of the cone

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

is

$$Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy = 0$$

where

$$A = bc - f^2, B = ca - g^2, C = ab - h^2,$$

$$F = gh - af, G = hf - bg, H = fg - ch.$$

Hence the reciprocal to $x^2 + z^2 - 2yz + 4zx = 0$ will be

$$\begin{aligned} & (0 \cdot 1 - (-1)^2) \cdot x^2 + (1 \cdot 1 - 2^2) \cdot y^2 + (1 \cdot 0 - 0^2) \cdot z^2 + \\ & + 2 \cdot (2 \cdot 0 - 1 \cdot (-1)) \cdot yz + 2 \cdot (0 \cdot (-1) - 0 \cdot 2) \cdot zx + 2 \cdot ((-1) \cdot 2 - 1 \cdot 0) \cdot zx \\ & = -x^2 - 3y^2 + 2yz - 4zx = 0. \end{aligned}$$

Answer: $-x^2 - 3y^2 + 2yz - 4zx = 0$.