# Answer on Question #45078 – Math – Analytic Geometry

**Question:**

Find the new equation of the conicoid $2x^2 + 3y^2 + 5z^2 - xy + z = 1$ when the coordinate system is transformed into a new system with the origin and with the coordinate axes having direction ratios 2,1,0; -1,2,5; 1,-2,1 with respect to the old system.

**Solution.**

Let denote axes of new coordinate system u,v,w. As a new system is with the origin and with the coordinate axes having direction ratios 2,1,0; -1,2,5; 1,-2,1 with respect to the old system, hence we can conclude that u=2x+y, v=x+2y+5z, w=x-2y+z.

Hence, the transformation matrix is

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -2 \\ 0 & 5 & 1 \end{pmatrix}.$$

So, the matrix of inverse transformation is

$$\begin{pmatrix} 3/7 & \frac{1}{7} & -\frac{1}{7} \\ -\frac{1}{28} & \frac{1}{14} & \frac{5}{28} \\ \frac{5}{28} & -\frac{5}{14} & \frac{3}{28} \end{pmatrix}.$$

Thus, $x = \frac{3}{7u} - \frac{1}{28}v + \frac{5}{28}w$, $y = \frac{1}{7}u + \frac{1}{14}v - \frac{5}{14}w$ and $z = -\frac{1}{7}u + \frac{5}{28}v + \frac{3}{28}w$.

Substituting this into the equation of the conicoid in the old system, we get

$$2x^2 + 3y^2 + 5z^2 - xy + z = 1$$

$$2(\frac{3}{7}u - \frac{1}{28}v + \frac{5}{28}w)^2 + 3(\frac{1}{7}u + \frac{1}{14}v - \frac{5}{14}w)^2 + 5(-\frac{1}{7}u + \frac{5}{28}v + \frac{3}{28}w)^2 - \left(\frac{3}{7}u - \frac{1}{28}v + \frac{5}{28}w\right)\left(\frac{1}{7}u + \frac{1}{14}v - \frac{5}{14}w\right) - \frac{1}{7}u + \frac{5}{28}v + \frac{3}{28}w = 1.$$

After simplification, we get

$$\frac{23\,u^2}{49} + \frac{141\,v^2}{784} + \frac{445\,w^2}{784} - \frac{55\,u\,v}{196} - \frac{5\,u\,w}{196} - \frac{5\,v\,w}{392} - \frac{u}{7} + \frac{5\,v}{28} + \frac{3\,w}{28} = 1$$

**Answer.** The new equation of the conicoid is

$$\frac{23\,u^2}{49} + \frac{141\,v^2}{784} + \frac{445\,w^2}{784} - \frac{55\,u\,v}{196} - \frac{5\,u\,w}{196} - \frac{5\,v\,w}{392} - \frac{u}{7} + \frac{5\,v}{28} + \frac{3\,w}{28} = 1$$