Answer on Question #45075 – Math – Analytic Geometry

Find the point of intersection of the line x/4=y=z-1 and the plane 2x+y+z=5.
Also find the angle between them.

Solution.

To find the point of intersection of the line

$$\frac{x}{4} = y = z - 1 \qquad (1)$$

and the plane

$$2x + y + z = 5, \qquad (2)$$

we solve the system of linear equations

$$\begin{cases} \frac{x}{4} = y = z - 1 \\ 2x + y + z = 5 \end{cases} \qquad (3)$$

Express variables x and z as functions of y on the basis of (1), we come to

$$x = 4y, z = y + 1.$$

Plug these expressions into the second equation (2) of the system (3):

$$2x + y + z = 5, \;\; 2 * 4y + y + y + 1 = 5,$$

Collect similar terms: $10y = 4$, divide both sides by 10 and obtain $y = \frac{4}{10}, y = \frac{2}{5}$.

Other coordinates of this point are $x = 4y = 4 * \frac{4}{10} = \frac{8}{5}, z = y + 1 = \frac{2}{5} + 1 = \frac{7}{5}$.

Thus, the point of intersection of the line and plane is $(x, y, z) = \left(\frac{8}{5}, \frac{2}{5}, \frac{7}{5}\right)$.

Vector $\boldsymbol{N} = (A, B, C) = (2,1,1)$ is the normal to the plane (2), vector

$\boldsymbol{u} = (l, m, n) = (4,1,1)$ is the direction vector of line (1).

The angle between the line and the plane is defined as the complementary angle θ of the angle φ between the direction vector $\boldsymbol{u}$ of the line and the normal $\boldsymbol{N}$ to the plane. For this angle, one has the formula $\sin(\theta) = \cos(\varphi) = \frac{Al+Bm+Cn}{\sqrt{A^2+B^2+C^2}\sqrt{l^2+m^2+n^2}}$

$= \frac{2*4+1*1+1*1}{\sqrt{2^2+1^2+1^2}\sqrt{4^2+1^2+1^2}} = \frac{10}{\sqrt{6*18}} = \frac{10}{6\sqrt{3}} = \frac{5}{3\sqrt{3}}$, hence $\theta = 1.295$ radians or $\theta = 74.21^{\circ}$