Problem.
Identify the conic x^2+xy+2y^2-2x-5=0. Also trace it.

Solution.
$A = 1, B = 1, C = 2.$ $B^2 - 4AC = 1^2 - 4 \cdot 1 \cdot 2 = -7 < 0,$ so it is ellipse.

$$x^2 + xy + 2y^2 - 2x - 5 = 2y^2 + xy + \frac{x^2}{8} + \frac{7x^2}{8} - 2x + \frac{8}{7} - \frac{8}{7} - 5$$

$$= \left(\sqrt{2}y + \frac{x}{2\sqrt{2}}\right)^2 + \left(\frac{\sqrt{7}x}{2\sqrt{2}} - \frac{2\sqrt{2}}{\sqrt{7}}\right)^2 - 5.$$

We obtain the conic $\left(\sqrt{2}y + \frac{x}{2\sqrt{2}}\right)^2 + \left(\frac{\sqrt{7}x}{2\sqrt{2}} - \frac{2\sqrt{2}}{\sqrt{7}}\right)^2 = \frac{43}{7}$ or $\frac{14}{43}\left(y + \frac{x}{4}\right)^2 + \frac{49}{344}\left(x - \frac{8}{7}\right)^2 = 1.$