Answer on Question #44957 – Math - Statistics and Probability

The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that at least 1 of them would have scored above 75?

Solution

Let event A=" at least 1 of 3 students would have scored above 75"

The complement of A is $\bar{A}$="none of 3 students would have scored above 75 "

$\bar{A} = B \cap C \cap D$ is intersection of three events *B*, *C*, *D*, where

B="the first student wouldn`t have scored above 75 "=

= "the first student would have scored less than 75";

C="the second student wouldn`t have scored above 75 "=

= "the second student would have scored less than 75 ";

D="the third student wouldn`t have scored above 75 "=

="the third student would have scored less than 75 ".

B, *C*, *D* are jointly statistically independent events.

Let X represents the marks obtained by a student. X is normally distributed with mean 65 and standard deviation 5.

Variable $Z = \frac{X-65}{5}$ has the standard normal distribution

x=75 gives $z = \frac{75-65}{5} = 2$.

Evaluate $P(B) = P(C) = P(D) = P(X < 75) = P\left(\frac{X-65}{5} < \frac{75-65}{5}\right) = P(Z < 2) = 0.9772$ (refer to statistical tables)

$$P(\bar{A}) = P(B \cap C \cap D) = P(B)P(C)P(D) = P(B)^3 = 0.9772^3 \approx 0.9931$$

Law of complement $P(A) = 1 - P(\bar{A}) \approx 1 - 0.9931 = 0.0069$.

Answer: 0.0069.