

Answer on Question #44956 – Math - Statistics and Probability

Find the probability that a person tossing 3 fair coins get either all heads or all tails for the second time on the fifth trial.

Solution

Let X be the number of trials (tossing) until get either all heads or all tails for the second time. Then X is a negative binomial random variable,

$$b(x; k, p) = \binom{x-1}{k-1} p^k q^{x-k}, x = k, k+1, k+2, \dots, k=2.$$

With,

$$p = P(\text{success}) = P(\text{all heads or all tails}) = \frac{2}{8} = \frac{1}{4}.$$

$$q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}.$$

$$P[X = 5] = b\left(5; 2, \frac{1}{4}\right) = \binom{4}{1} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3 = \frac{27}{256}.$$

Answer: $\frac{27}{256}$.