Answer on Question #44952 – Math - Statistics and Probability

Problem.

A and B alternatively throw a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is 30/61.

Solution.

A could win the game when he rolls a 6 at the 1-st throw or when he rolls 6 at the k-th throw ($k > 1$) and in the first $k-1$ throws A does not roll a 6 and in the first $k-1$ throws B does not roll a 7.

There are $6 \cdot 6 = 36$ different combination, that could be obtained in the each throw. There are 5 possibilities to obtain 6 ($1+5=2+4=3+3=4+2=5+1=6$) and 6 possibilities to obtain 7 ($1+6=2+5=3+4=4+3=5+2=6+1=7$). Therefore

$$P(A \text{ roll a 6 at some throw}) = \frac{5}{36},$$

$$P(A \text{ does not roll a 6 at some throw}) = 1 - \frac{5}{36} = \frac{31}{36},$$

$$P(B \text{ roll a 7 at some throw}) = \frac{7}{36},$$

$$P(B \text{ does not roll a 7at some throw}) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6},$$

$$P(A \text{ does not roll a 6 at some throw and then } B \text{ does not roll a 7 at some throw}) = \frac{31}{36} \cdot \frac{5}{6} = \frac{155}{216}.$$

Hence

$$P(A \text{ wins at the } k - \text{th throw}) = \left(\frac{155}{216}\right)^{k-1} \cdot \frac{5}{36}$$

The total probability that A wins equals

$$P(A \text{ wins}) = \sum_{k=0}^{\infty} P(A \text{ wins at the } k - \text{th throw}) = \sum_{k=0}^{\infty} \left(\frac{155}{216}\right)^{k-1} \cdot \frac{5}{36} = \frac{\frac{5}{36}}{1 - \frac{155}{216}} = \frac{30}{61},$$

as the sum of infinite geometric progression.