Answer on Question #44951 – Math – Statistics and Probability

Suppose that the joint density of X and Y is given by

$\mathrm{f}(x, y) = e^{-\left(\frac{x}{y}\right)} \cdot e^{-y}$,$0 < x < \infty, 0 < y < \infty$ and is o otherwise. Find $P(X > 1 \mid Y = y)$.

Solution

$$P(X > 1 \mid Y = y) = \int_1^{\infty} \mathrm{f}_{X|Y}(x|y)dx.$$

$$\mathrm{f}_{X|Y}(x|y) = \frac{\mathrm{f}(x, y)}{\mathrm{f}_{\mathrm{Y}}(y)} = \frac{e^{-\left(\frac{x}{y}\right)} \cdot e^{-y}}{\int_0^{\infty} e^{-\left(\frac{x}{y}\right)} \cdot e^{-y} dx} = \frac{e^{-\left(\frac{x}{y}\right)} \cdot e^{-y}}{e^{-y} \int_0^{\infty} e^{-\left(\frac{x}{y}\right)} dx} = \frac{e^{-\left(\frac{x}{y}\right)}}{y}.$$

$$P(X > 1 \mid Y = y) = \int_1^{\infty} \mathrm{f}_{X|Y}(x|y)dx = \int_1^{\infty} \frac{e^{-\left(\frac{x}{y}\right)}}{y} dx = \frac{1}{y} \int_1^{\infty} e^{-\left(\frac{x}{y}\right)} dx = \frac{1}{y}\left(y e^{-\frac{1}{y}}\right) = e^{-\frac{1}{y}}.$$