Answer on Question #44950 – Math – Statistics and Probability

Question. The random variable X has probability density function $f(x) = ax + bx^2, 0 < x < 1$. If $E(X) = 0.6$, find

a) $P(X < 1/2)$

b) $Var(X)$

Solution. First of all we shall find a and b. Since $f(x) = ax + bx^2, 0 < x < 1$ is the probability density function, then $\int_0^1 f(x)dx = 1$. $\int_0^1 (ax + bx^2)dx = \frac{a}{2}x^2 + \frac{b}{3}x^3|_0^1 = \frac{a}{2} + \frac{b}{3}$. The first condition: $\frac{a}{2} + \frac{b}{3} = 1$.

$E(X) = \int_0^1 x(ax + bx^2)dx = \int_0^1 (ax^2 + bx^3)dx = \frac{a}{3}x^3 + \frac{b}{4}x^4|_0^1 = \frac{a}{3} + \frac{b}{4}$. The second condition: $\frac{a}{3} + \frac{b}{4} = \frac{3}{5}$. We have the next linear system:

$$\begin{cases} \frac{a}{2} + \frac{b}{3} = 1 \\ \frac{a}{3} + \frac{b}{4} = \frac{3}{5} \end{cases} \Leftrightarrow \begin{cases} 3a + 2b = 6 \\ 4a + 3b = \frac{36}{5} \end{cases} \Leftrightarrow \begin{cases} 9a + 6b = 18 \\ -8a - 6a = -\frac{72}{5} \end{cases} \Leftrightarrow \begin{cases} a = \frac{18}{5} \\ b = -\frac{12}{5} \end{cases}.$$

The density function has the form: $f(x) = \frac{18}{5}x - \frac{12}{5}x^2, 0 < x < 1$.

a) $P(X < 1/2) = \int_0^{1/2} \left(\frac{18}{5}x - \frac{12}{5}x^2\right) dx = \frac{9}{5}x^2 - \frac{4}{5}x^3|_0^{1/2} = \frac{9}{20} - \frac{2}{20} = \frac{7}{20}$.

b) $E(X^2) = \int_0^1 x^2 \left(\frac{18}{5}x - \frac{12}{5}x^2\right) dx = \int_0^1 \left(\frac{18}{5}x^3 - \frac{12}{5}x^4\right) dx = \frac{9}{10} - \frac{12}{25} = \frac{21}{50}$.

$$Var(X) = E(X^2) - [E(X)]^2 = \frac{21}{50} - \frac{9}{25} = \frac{3}{50}.$$

Answer. a) $P(X < 1/2) = \frac{7}{20}$

b) $Var(X) = \frac{3}{50}$.