Answer on Question #44949 – Math - Statistics and Probability

A communication channel transmits the digits 0 and 1. However due to static noise, the digit transmitted is incorrectly received with probability 0.2. Suppose we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0. And 11111 instead of 1. If the receiver of the message uses majority decoding , what is the probability that the message will be incorrectly decoded. (By majority decoding we mean that the message is decoded as 0 if there are At least 3 zeros in the message received and as 1 otherwise.)

Solution

Since we have 5 bits, corruption will only occur if at least 3 bits are incorrect for the same block.

I will call an incorrectly transmitted bit a "success" since that's what we are watching out for.

The binomial distribution gives the probability of exactly m successes in n trials where the probability of each individual trial succeeding is p. With this method, you want the following:

Binomial at (m=3,n=5,p=0.2) for probability of corruption due to exactly 3 fails,

Binomial at (m=4,n=5,p=0.2) for probability of corruption due to exactly 4 fails,

Binomial at (m=5,n=5,p=0.2) for probability of corruption due to exactly 5 fails.

The probability of this is

$$P = \binom{5}{3} p^3 (1-p)^{5-3} + \binom{5}{4} p^4 (1-p)^{5-4} + \binom{5}{5} p^5 (1-p)^{5-5}.$$

$$\binom{5}{3} = \frac{5!}{3!\,(5-3)!} = 10; \binom{5}{4} = \frac{5!}{4!\,(5-4)!} = 5; \binom{5}{5} = \frac{5!}{5!\,(5-5)!} = 1.$$

$$P = 10(0.2)^3(1-0.2)^2 + 5(0.2)^4(1-0.2)^1 + (0.2)^5 = 0.05792.$$

Answer: $\mathbf{0.05792}$.