

Answer on Question #44929 – Math - Linear Algebra

Problem.

Let $P^3 = \{ax^3 + bx^2 + cx + d \mid a, b, c, d \in \mathbb{R}\}$. Check whether $f(x) = x^2 + 2x + 1$ is in $[S]$, the subspace of P^3 generated by $S = \{3x^2 + 1, 2x^2 + x + 1\}$.

If $f(x)$ is in $[S]$, write f as a linear combination of elements in S .

If $f(x)$ is not in $[S]$, find another polynomial $g(x)$ of degree at most two such that $f(x)$ is in the span of $S \cup \{g(x)\}$.

Also write f as a linear combination of elements in $S \cup \{g(x)\}$.

Solution.

$x^2 + 2x + 1 = 2(2x^2 + x + 1) - (3x^2 + 1)$, so $f(x) \in [S]$.