Answer on Question #44928 – Math – Linear Algebra

Question. Let $V = \{(a, b, c, d) \in \mathbb{R}^4 : a + b + 2c + 2d = 0\}$ and $W = \{(a, b, c, d) \in \mathbb{R}^4 : a = -b, c = -d\}$. Find the dimensions of V and W.

Solution. Rewrite the V in the next form: $V = \{(-b - 2c - 2d, b, c, d)\}$. Find the basis of V: $v_1 = (-1, 1, 0, 0), v_2 = (-2, 0, 1, 0), v_3 = (-2, 0, 0, 1)$. Check that v_1, v_2, v_3 are linearly independent.

Let $\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 = \bar{0} \Leftrightarrow (-\lambda_1, \lambda_1, 0, 0) + (-2\lambda_2, 0, \lambda_2, 0) + (-2\lambda_3, 0, 0, \lambda_3) = \bar{0} \Leftrightarrow$

$\Leftrightarrow (-\lambda_1 - 2\lambda_2 - 2\lambda_3, \lambda_1, \lambda_2, \lambda_3) = \bar{0} \Leftrightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = 0 \\ \lambda_3 = 0 \end{cases} \Rightarrow v_1, v_2, v_3$ are lineary independent.

Check that $\forall v \in V$ can be represented in the next form: $v = \lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3$. Let $v = (-b - 2c - 2d, b, c, d) \in V$. Then $\begin{cases} \lambda_1 = b \\ \lambda_2 = c \\ \lambda_3 = d \end{cases}$. $bv_1 + cv_2 + dv_3 = (-b - 2c - 2d, b, c, d) == v$.
So dimension of V is equal to 3.

Rewrite the W in the next form: $W = \{(a, -a, c, -c)\}$. Find the basis of W: $w_1 = (1, -1, 0, 0), w_2 = (0, 0, 1, -1)$. Check that w_1, w_2 are linearly independent.

Let $\lambda_1 w_1 + \lambda_2 w_2 = \bar{0} \Leftrightarrow (\lambda_1, -\lambda_1, 0, 0) + (0, 0, \lambda_2, -\lambda_2) = \bar{0} \Leftrightarrow (\lambda_1, -\lambda_1, \lambda_2, -\lambda_2) = \bar{0} \Leftrightarrow$

$\Leftrightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = 0 \end{cases} \Rightarrow w_1, w_2$ are lineary independent.

Check that $\forall w \in W$ can be represented in the next form: $w = \lambda_1 w_1 + \lambda_2 w_2$. Let $w = (a, -a, c, -c) \in W$. Then $\begin{cases} \lambda_1 = a \\ \lambda_2 = c \end{cases}$. $aw_1 + cw_2 = (a, -a, c, -c) = w$. So dimension of W is equal to 2.

Answer. $dimV = 3, dimW = 2$.