Answer on Question #44927 – Math – Linear Algebra

Question. Let $V = \{(a,b,c,d) \in \mathbb{R}^4 : a + b + 2c + 2d = 0\}$ and $W = \{(a,b,c,d) \in \mathbb{R}^4 : a = -b, c = -d\}$. Check that V and W are the vector spaces. Further, check that W is a subspace of V.

Solution. We shall prove that V and W are the subspaces of $\mathbb{R}^4$. We shall use the next criterion of subspace: the subset W of linear space V is a subspace of $V \Leftrightarrow \begin{cases} (\bar{a} + \bar{b}) \in W \ \forall \bar{a}, \bar{b} \in W \\ \lambda\bar{a} \in W \ \forall \lambda \in \mathbb{R}, \forall \bar{a} \in W \end{cases}$.

Let $\bar{a} = (a_1, b_1, c_1, d_1) \in V, \bar{b} = (a_2, b_2, c_2, d_2) \in V$. Then $\bar{a} + \bar{b} = (a_1 + a_2, b_1 + b_2, c_1 + {} + c_2, d_1 + d_2)$.

$a_1 + a_2 + b_1 + b_2 + 2(c_1 + c_2) + 2(d_1 + d_2) = (a_1 + b_1 + 2c_1 + 2d_1) + (a_2 + b_2 + 2c_2 + {} + 2d_2) = 0 + 0 = 0 \Rightarrow (\bar{a} + \bar{b}) \in V$.

$\lambda\bar{a} = (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1)$. Then $\lambda a_1 + \lambda b_1 + 2\lambda c_1 + 2\lambda d_1 = \lambda(a_1 + b_1 + 2c_1 + 2d_1) = \lambda \cdot 0 = {} = 0 \Rightarrow \lambda\bar{a} \in V$. So V is a subspace of $\mathbb{R}^4 \Rightarrow V$ is a vector space.

Let $\bar{a} = (a_1, -a_1, b_1, -b_1) \in W, \bar{b} = (a_2, -a_2, b_2, -b_2) \in W$. Then $\bar{a} + \bar{b} = (a_1 + a_2, -a_1 - {} - a_2, b_1 + b_2, -b_1 - b_2)$. Obviously $(\bar{a} + \bar{b}) \in W$.

$\lambda\bar{a} = (\lambda a_1, -\lambda a_1, \lambda b_1, -\lambda b_1)$. Obviously $\lambda\bar{a} \in W$. So W is a subspace of $\mathbb{R}^4 \Rightarrow W$ is a vector space.

Let $\bar{x} = (a, -a, c, -c) \in W$. Since $a + (-a) + 2c + (-2c) = 0 \Rightarrow \bar{x} \in V \Rightarrow W$ is a subspace of V.

Answer. V and W are the vector spaces, W is a subspace of V.