

Answer on Question #44926 – Math - Linear Algebra

Problem.

There is no matrix which is Hermitian as well as Unitary.

Solution.

The statement is false.

Let $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{bmatrix}$ (where $0 \leq \alpha \leq 2\pi$).

$\overline{A^T} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{bmatrix} = A$, so A is Hermitian.

$\overline{A^T}A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, so A is unitary.

Hence A is Hermitian as well as unitary.