Answer on Question #44925 – Math - Linear Algebra

Problem.

State if the following statements are true and which are false? Justify your answer with a short proof or a counterexample.

No skew-symmetric matrix is diagonalisable.

Solution.

The statement is false.

Let $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} -\frac{i}{2} & \frac{1}{2} \\ \frac{i}{2} & \frac{1}{2} \end{bmatrix}$. Then $Q^{-1} = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}$ (as $\det Q = -\frac{i}{2}$).

Hence

$$D = QAQ^{-1} = \begin{bmatrix} -\frac{i}{2} & \frac{1}{2} \\ \frac{i}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{i}{2} & \frac{1}{2} \\ \frac{i}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2+i & 2-i \\ 1-2i & 1+2i \end{bmatrix} = \begin{bmatrix} 1-2i & 0 \\ 0 & 1+2i \end{bmatrix}.$$

Therefore A is diagonalisable.