Answer on Question #44924 – Math – Linear Algebra:

State if the following statements are true and which are false? Justify your answer with a short proof or a counterexample.

If a linear operator is diagonalisable, its minimal polynomial is the same as the characteristic polynomial.

Solution.

It’s false. Counterexample:

$$L: \mathbb{R}^2 \to \mathbb{R}^2, L\begin{pmatrix} x \\ y \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix}, A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix};$$

Prove that the minimal polynomial of L is $f(x) = x - 2$.

$$f(A) = A - 2E = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} - 2 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 0;$$

$f(A) = 0$ and f is of the degree 1. So, f is minimal polynomial.

Compute it’s characteristic polynomial $\chi(x)$.

$$\chi(x) = \det(xE - A) = \begin{vmatrix} x-2 & 0 \\ 0 & x-2 \end{vmatrix} = (x-2)^2 \neq f(x).$$