Problem.

State if the following statements are true and which are false? Justify your answer with a short proof or a counterexample.

If the characteristic polynomial of a linear transformation is (x-1)(x-2), its minimal polynomial is x-1 or x-2.

Solution.

The statement is false.

Suppose that $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ is a matrix of linear transformation. The characteristic polynomial of a linear transformation is $(x-1)(x-2)$, as

$\det \begin{bmatrix} 1-x & 0 \\ 0 & 2-x \end{bmatrix} = (1-x)(2-x) = (x-1)(x-2).$

$x-1$ isn't minimal polynomial of $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, as $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$

$x-2$ isn't minimal polynomial of $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, as $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$