Answer on Question #44921 – Math - Linear Algebra

The row-reduced echelon form of an invertible matrix is the identity matrix.

Solution

Some definitions:

In linear algebra, t**he identity matrix** or unit matrix of size n is the n × n square matrix with ones on the main diagonal and zeros elsewhere. It is denoted by In, or simply by I if the size is immaterial or can be trivially determined by the context. (In some fields, such as quantum mechanics, the identity matrix is denoted by a boldface one, 1; otherwise it is identical to I.) Some mathematics books use U and E to represent the Identity Matrix (meaning "Unit Matrix" and the German word "Einheitsmatrix", respectively), although I is considered more universal.

$$I_1 = \begin{bmatrix} 1 \end{bmatrix},\ I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \cdots,\ I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

In linear algebra, Gaussian elimination (also known as **row reduction**) is an algorithm for solving systems of linear equations. It is usually understood as a sequence of operations performed on the associated matrix of coefficients. This method can also be used to find the rank of a matrix, to calculate the determinant of a matrix, and to calculate the inverse of an invertible square matrix.

In linear algebra, a matrix is in **echelon form** if it has the shape resulting of a Gaussian elimination. Row echelon form means that Gaussian elimination has operated on the rows andcolumn echelon form means that Gaussian elimination has operated on the columns. In other words, a matrix is in column echelon form if its transpose is in row echelon form. Therefore only row echelon forms are considered in the remainder of this article. The similar properties of column echelon form are easily deduced by transposing all the matrices.

Specifically, a matrix is in row echelon form if

1. All nonzero rows (rows with at least one nonzero element) are above any rows of all zeroes (all zero rows, if any, belong at the bottom of the matrix).

2. The leading coefficient (the first nonzero number from the left, also called the pivot) of a nonzero row is always strictly to the right of the leading coefficient of the row above it (some texts add the condition that the leading coefficient must be 1.).

3. All entries in a column below a leading entry are zeroes (implied by the first two criteria).

This is an example of a 3×5 matrix in row echelon form:

$$\begin{bmatrix} 1 & a_0 & a_1 & a_2 & a_3 \\ 0 & 0 & 2 & a_4 & a_5 \\ 0 & 0 & 0 & 1 & a_6 \end{bmatrix}$$

A matrix is in reduced row echelon form (also called row canonical form) if it satisfies the following conditions:

1. It is in row echelon form.
2. Every leading coefficient is 1 and is the only nonzero entry in its column.

The reduced row echelon form of a matrix may be computed by Gauss–Jordan elimination. Unlike the row echelon form, the reduced row echelon form of a matrix is unique and does not depend on the algorithm used to compute it.

This is an example of a matrix in reduced row echelon form:

$$\begin{bmatrix} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & 0 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{bmatrix}$$

ANSWER: Note that this does not always mean that the left of the matrix will be an identity matrix, as this example shows.