Answer on Question #44854 – Math - Statistics and Probability

A firm has classified its customers in two ways: (1) according to whether the account is overdue and (2) whether the account is new (less than 12 months) or old. To acquire information about which customers are paying on time and which are overdue, a random sample of 292 customer accounts was drawn. Each was categorized as either a new account or an old account, and whether the customer has paid or is overdue. The results are summarized next.

	New account	Old account
Sample size	83	209
Overdue account	12	49

Is there enough evidence at the 5% significance level to infer that new and old accounts are different with respect to overdue accounts?

Solution

Hypotheses are $H_0: p_1 - p_2 = 0$, $H_a: p_1 - p_2 \neq 0$.

$$\widehat{p_1} = \frac{12}{83} = 0.145; \; \widehat{p_2} = \frac{49}{209} = 0.234$$

$$\hat{p} = \frac{n_1\widehat{p_1} + n_2\widehat{p_2}}{n_1 + n_2} = \frac{83 \cdot 0.145 + 209 \cdot 0.234}{83 + 209} = 0.209$$

Test statistics is $z = \dfrac{\widehat{p_1} - \widehat{p_2}}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \dfrac{0.145 - 0.234}{\sqrt{0.209(1-0.209)\left(\frac{1}{83}+\frac{1}{209}\right)}} = -1.69.$

$$p - value = 2 \cdot P(Z < -1.69) = 2 \cdot (0.0455) = 0.0910 > 0.05.$$

Conclusion. There is not enough evidence to conclude that new and old accounts are different with respect to overdue accounts.