Answer on Question #44851 – Math – Statistics and Probability

A number of restaurants feature a device that allows credit card users to swipe their cards at the table. It allows the user to specify a percentage or a dollar amount to leave as a tip. In an experiment to see how it works, a random sample of credit card users was drawn. Some paid the usual way, and some used the new device. The percent left as a tip was recorded and listed below. Can we infer that users of the device leave larger tips?

Usual: 10.3 15.2 13.0 9.9 12.1 13.4 12.2 14.9 13.2 12.0

Device: 13.6 15.7 12.9 13.2 12.9 13.4 12.1 13.9 15.7 15.4 17.4

Solution

First, we need to state the null hypothesis and an alternative hypothesis.

Null hypothesis: $\mu1 - \mu2 = 0$ (there is no difference between the percent left as a tip)

Alternative hypothesis: $\mu1 - \mu2 < 0$ (users of the device leave larger tips, the mean for the device leave tips is more than in the usual manner).

We note that these hypotheses constitute a one-tailed test. The null hypothesis will be rejected if the mean difference between sample means is too small. For this analysis, the significance level is 0.05. Using sample data, we will conduct a two-sample t-test.

We had determined the following given data:

$$\mathrm{n}_1 = 10, \mathrm{n}_2 = 11, \bar{\mathrm{x}}_1 = 12.62, \bar{\mathrm{x}}_2 = 14.2, \mathrm{s}_1 = 1.718, \mathrm{s}_2 = 1.614$$

Sample standard deviations are approximately equal.

If we do not know s then we use the pooled standard deviation.

$$\mathrm{S_p} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Substitute the values into the formula.

$$\mathrm{S_p} = \sqrt{\frac{(10-1)\cdot(1.718)^2 + (11-1)\cdot(1.614)^2}{10+11-2}} = \sqrt{\frac{26.556 + 26.060}{19}} = \sqrt{\frac{52.616}{19}} = 1.664$$

Then calculate the t-score (t) defined by the following equation:

$$\mathrm{t} = \frac{[(\bar{\mathrm{x}}_1 - \bar{\mathrm{x}}_2) - \mathrm{d}]}{\mathrm{S_p}\sqrt{\frac{1}{\mathrm{n}_1} + \frac{1}{\mathrm{n}_2}}} = \frac{12.62 - 14.2 - 0}{1.664 \cdot \sqrt{\frac{1}{10} + \frac{1}{11}}} = \frac{-1.58}{0.727} = -2.173$$

The number of degrees of freedom of the test statistic is

$$df = n_1 + n_2 - 2 = 10 + 11 - 2 = 19$$

Because this is an one-tailed test, the alpha level (0.05) is not divided by two. The p-value is equal to 0.0216. Thus, we can conclude that the result is significant at $p < 0.05$. Since the P-value (0.0216) is less than the significance level (0.05), we cannot accept the null hypothesis, this means that we can confirm that there is no difference between the percent left as a tip. Thus, we can state that there is a sufficient evidence to make a conclusion that users of the device leave larger tips.