Answer on Question #44849 - Math - Statistics and probability

Every month a clothing store conducts an inventory and calculates losses from theft. The store would like to reduce the losses and is considering two methods. The first is to hire a security guard, and the second is to install cameras. To help decide which method to choose, the manager hired a security guard for 6 months. During the next 6-month period, the store installed cameras. The monthly losses were recorded are listed here. The manager decided that because the cameras were cheaper than the guard, he would install the cameras unless there was enough evidence to infer that the guard was better. What should the manager do?

Security guard: 355 284 401 398 477 254
Cameras: 486 303 270 386 411 435

Answer

Let a=0.05.
Let's define μ_d as losses in the case of guard minus losses in the case of cameras.
The null hypothesis contains "equal" sign ("=" or "≥" or "≤"), the alternative hypothesis is the complement to the null hypothesis.
The null hypothesis: means are equal, the monthly losses are equal in the case of guard and in the case of cameras.
H_0: $\mu_d \geq 0$
The alternative hypothesis: the guard is better, i.e. losses in the case of guard are less.
H_a: $\mu_d < 0$
As alternative hypothesis contain "<" sign, we will use left-tailed test.

We will use t-distribution as number of observations is less than 30 and 95% confidence level, i. e. $\alpha = 0.05$.

We assume that variances are not equal as we do not know them, so df = 5. Critical value at 5 degrees of freedom: t_c= -2.015.

Standardized test statistics is:
$$t = \frac{(\bar{x_1} - \bar{x_2}) - \mu_d}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{361.5 - 381.833}{\sqrt{\frac{82.265^2}{6} + \frac{81.568^2}{6}}} = -0.430$$

As the test statistic does not lie in the rejection region (t < -2.015), we does not have enough evidence to reject the null hypothesis that means are equal.

At 95% confidence level, we cannot reject the null hypothesis, so we can conclude that the mean losses are equal in the case of guard and in the case of cameras.
Thus, manager could buy cheaper cameras.

R code:
d1 <- c(355, 284, 401, 398, 477, 254)
d2 <- c(486, 303, 270, 386, 411, 435)
t.test(d1,d2,level=0.95,alternative="less",var.equal=F)