

Answer on Question #44746-Math-Statistics and Probability

The table below shows the time distribution of fifty depositors to finish a transaction with bank personnel.

Time Number of Depositors (f) (nearest minute)

23-25 4

20-22 6

17-19 16

14-16 8

11-13 8

8-10 6

5-7 2

Determine the values of:

a) We can estimate the Mean by using the midpoints.

| Midpoint (x) | Frequency (f) | $f \cdot x$ |
|--------------|---------------|-------------|
| 24 | 4 | 96 |
| 21 | 6 | 126 |
| 18 | 16 | 288 |
| 15 | 8 | 120 |
| 12 | 8 | 96 |
| 9 | 6 | 54 |
| 6 | 2 | 12 |
| Totals: | 50 | 792 |

So an estimate of the mean is

$$\text{Estimated Mean} = \frac{792}{50} = 15.8$$

b) if we need to estimate a single Median value we can use this formula:

$$\text{Estimated Median} = L + \frac{\frac{n}{2} - cf_b}{f_m} \cdot w$$

where:

L is the lower class limit of the group containing the median,

n is the total number of data,

cf_b is the cumulative frequency of the groups before the median group,

f_m is the frequency of the median group,

w is the group width.

The median group is 17-19.

For our example:

$$L = 16.5, n = 50, cf_b = 4 + 6 = 10, f_m = 16, w = 3.$$

$$\text{Estimated Median} = 16.5 + \frac{\frac{50}{2} - 10}{16} \cdot 3 = 19.3.$$

c) We can easily identify the modal group (the group with the highest frequency), which is 17-19.

We can estimate the Mode using the following formula:

$$\text{Estimated Mode} = L + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})}$$

where:

L is the lower class limit of the modal group

f_{m-1} is the frequency of the group before the modal group

f_m is the frequency of the modal group

f_{m+1} is the frequency of the group after the modal group

w is the group width.

In this example:

$$L = 16.5, f_m = 16, f_{m-1} = 6, f_{m+1} = 8, w = 3.$$

$$\text{Estimated Mode} = 16.5 + \frac{16 - 6}{(16 - 6) + (16 - 8)} = 17.0.$$

d) First quartile lies in $\frac{50}{4} = 12.5$ place. And exact first quartile is

$$\text{first quartile} = L + \frac{\frac{N}{4} - cf}{f} \cdot w$$

where L is lower limit of first quartile class, f is frequency of first quartile class, cf is cumulative frequency of pre-first quartile class, w is size of first quartile class, N is total numbers of items.

In this example (first quartile class is 17-19):

$$L = 16.5, f = 16, N = 50, cf = 4 + 6 = 10, w = 3.$$

$$\text{first quartile} = 16.5 + \frac{\frac{50}{4} - 10}{16} \cdot 3 = 17.0.$$

e) Fourth quartile lies in 50 place. And exact first quartile is

$$\text{fourth quartile} = L + \frac{N - cf}{f} \cdot w$$

where L is lower limit of fourth quartile class, f is frequency of fourth quartile class, cf is cumulative frequency of pre-fourth quartile class, w is size of fourth quartile class, N is total numbers of items.

In this example (fourth quartile class is 5-7):

$$L = 5, f = 2, N = 50, cf = 48, w = 3.$$

$$\text{fourth quartile} = 5 + \frac{50 - 48}{2} \cdot 3 = 8.$$

f) kth percentile is

$$\text{kth percentile} = L + \frac{\frac{kN}{100} - cf}{f} \cdot w$$

where L is lower limit of P_k class, f is frequency of P_k class, cf is cumulative frequency of the class just preceding P_k class, w is size of P_k class, N is total numbers of items.

In this example (first quartile class is 11-13):

$$L = 10.5, f = 8, N = 50, cf = 34, w = 3.$$

$$80\text{th percentile} = 10.5 + \frac{\frac{80 \cdot 50}{100} - 34}{8} \cdot 3 = 12.7.$$