Answer on Question #44742 - Math - Algebra

Given the quadratic function y=ax²+bx+c, the maximum value is a²+4 at x=1, and the graph passes through point (3, 1). Find the values of the constants a, b and c.

Solution.

$$y = ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}.$$

y has maximum when $x = -\frac{b}{2a}$ and $a < 0$,

so $1 = -\frac{b}{2a} \rightarrow b = -2a$.

If $y(1) = a^2 + 4$ than $a + b + c = a^2 + 4$.

If the graph passes through point (3, 1) than $1 = 9a + 3b + c$.

So, we have 3 equations to find a, b and c:

$$\begin{cases} b = -2a \\ a + b + c = a^2 + 4 \\ 9a + 3b + c = 1 \end{cases}$$

Substitute $b = -2a$ into the second and the third equations.

$$\begin{cases} b = -2a \\ a - 2a + c = a^2 + 4 \\ 9a + 3(-2a) + c = 1 \end{cases}$$

$$\begin{cases} b = -2a \\ c - a = a^2 + 4 \\ 3a + c = 1 \end{cases}$$

From the third equation we conclude $c = 1 - 3a$ and substitute it into the second equation

$$\begin{cases} b = -2a \\ 1 - 3a - a = a^2 + 4 \\ c = 1 - 3a \end{cases}$$

$$\begin{cases} b = -2a \\ a^2 + 4a + 3 = 0 \\ c = 1 - 3a \end{cases}$$

Equation $a^2 + 4a + 3 = 0$ has two solutions: $a = -1$ and $a = -3$, which satisfy condition $a < 0$.

Finally, $b = -2a$ and $c = 1 - 3a$.

This system has two solutions:

$a = -1$, $b = 2$, $c = 4$

and

$a = -3$, $b = 6$, $c = 10$.

Answer: $(a, b, c) = (-1, 2, 4)$ or $(a, b, c) = (-3, 6, 10)$.