# Answer on Question #44695 – Math - Linear Algebra

Give some detail explanation on Pseudo inverse matrix??

## Solution

Pseudoinverse matrix – is a generalization of the inverse matrix in mathematics, particularly in linear algebra.

Pseudoinverse satisfies the following criteria:

- $AA^{+}A = A$ ($AA^{+}$ or $A^{+}A$ is not necessarily equal to the identity matrix);
- $(AA^{+})^{*} = AA^{+}$ (meaning that $AA^{+}$ - Hermitian matrix);
- $A^{+}AA^{+} = A^{+}$;
- $(A^{+}A)^{*} = A^{+}A$ ($A^{+}A$ - also Hermitian matrix);

where $A^{*}$ - Hermitian-conjugate matrix to the matrix A.

**Calculation**

- With A = BC schedule

Let r - rank matrix A size $m \times n$. Then A can be represented as A = BC, where B - matrix of size $m \times r$, C - matrix of size $r \times n$. Then

- $A^{+} = C^{*}(CC^{*})^{-1}(B^{*}B)^{-1}B^{*}$.

or

- $A^{+} = C^{*}(B^{*}AC^{*})^{-1}B^{*}$

- where $(CC^{*})^{-1}(B^{*}B)^{-1} = (B^{*}BCC^{*})^{-1} = (B^{*}AC^{*})^{-1}$ - a smaller matrix of size $r \times r$.

- Using QR decomposition

A matrix represented as A = QR, where Q - unitary matrix, $Q^{*}Q = QQ^{*} = I$, and R - upper triangular matrix. Then

- $A^{*}A = (QR)^{*}(QR) = R^{*}Q^{*}QR = R^{*}R$,
- $A^{+} = (R^{*}R)^{+}A^{*}$

**Properties**

- Pseudoinverse matrix always exists and is unique.
- Pseudoinverse matrix is equal to zero its transposition.
- Pseudoinverse is reversible to himself:

- $(A^{+})^{+} = A$.

- Pseudoinverse commutes with transposition, Hermitian coupling and coupling:

- $(A^T)^+ = (A^+)^T, \qquad (\overline{A})^+ = \overline{A^+}, \qquad (A^*)^+ = (A^+)^*.$

-Pilot matrix equals its rank to pseudoinverse:

- $\mathrm{rank}\ A^+ = \mathrm{rank}\ A$

- Pseudoinverse matrix product of A by a scalar $\alpha$ is the product of the matrix $A^+$ on inverse number $\alpha^{-1}$:
$(\alpha A)^+ = \alpha^{-1}\; A^+, \quad \forall \alpha \ne 0.$

-If already known matrix $(A^* A)^+$ or matrix $(AA^*)^+$, they can be used to calculate $A^+$:

- $A^+ = (A^* A)^+\; A^*$
- $A^+ = A^*\; (AA^*)^+.$

-Matrix $A^+ A,\; AA^+$ - is the orthogonal projection-matrices.
-If the matrix $A_i$ formed from the matrix $A$ by inserting another zero row / column in the i-th position, then $A_i^+$ will be created with $A^+$ by adding a zero column / row in the i-th position.