Answer on Question #44690 - Math - Statistics and Probability

A light bulb manufacturer guarantees that the mean life of a certain type of light bulb is at least 875 hours. A random sample of 58 light bulbs has a mean life of 852 hours with a standard deviation of 95 hours. Do you have enough evidence to reject the manufacturer's claim?
Use $\alpha = 0.09$.

a. Identify the null hypothesis and alternative hypothesis.
b. Identify the critical value (s). Use a comma to separate answers as needed.
c. Identify the standardized test statistic.
d. Decide whether to reject or fail to reject the null hypothesis.

Solution:

a. Identify the null hypothesis and alternative hypothesis:
The null hypothesis contains "equal" sign ("=" or "≥" or "≤"), the alternative hypothesis is the complement to the null hypothesis. The claim is "the mean life of a certain type of light bulb is at least 875 hours". As the claim contains "≥" sign, it is null hypothesis.
H_0: $\mu \geq 875$

Alternative hypothesis:
The alternative hypothesis is the complement: "the mean life of a certain type of light bulb is less than 875 hours".
H_a: $\mu < 875$

b. Identify the critical value (s). Use a comma to separate answers as needed
As the alternative hypothesis contains "<" sign, the test is left-tailed. Using $\alpha = 0.09$, we obtain critical value from standard table of normal distribution:

$z_0 = -1.34$

Thus, the rejection region for the test statistic is $z < -1.34$.

c. Identify the standardized test statistic.

The test statistic is:

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{852 - 875}{95/\sqrt{58}} = -1.84$$

d. Decide whether to reject or fail to reject the null hypothesis.

As $-1.84 < -1.34$, we should reject the null hypothesis, so we do not have enough evidence to support the manufacturer's claim.