Answer on Question #44654 - Math - Functional Analysis Find the range of function

$$f(x) = x^3 - \frac{1}{x} - 1$$

Firstly, let's find the derivative.

$$f'(x) = 3x^2 + \frac{1}{x^2} \ge 2\sqrt{3},$$

due to Cauchy inequality. Hence, function is increasing on $(-\infty, 0)$ and $(0, +\infty)$. Let's consider the second case.

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} (x^3 - \frac{1}{x} - 1) = -\infty,$$

due to the fact that x^3 and -1 are continuous in 0.

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} (x^3 - \frac{1}{x} - 1) = +\infty,$$

due to the fact that $-\frac{1}{x} \to 0(x \to \infty)$. Because of the fact that

$$f(x) = x^3 - \frac{1}{x} - 1$$

is continuous on $(0, +\infty)$, it reaches every value from lower to upper bound, therefore, it reaches every value in $(-\infty, +\infty)$