Answer on Question \#44654-Math - Functional Analysis
Find the range of function

$$
f(x)=x^{3}-\frac{1}{x}-1
$$

Firstly, let's find the derivative.

$$
f^{\prime}(x)=3 x^{2}+\frac{1}{x^{2}} \geq 2 \sqrt{3},
$$

due to Cauchy inequality. Hence, function is increasing on $(-\infty, 0)$ and $(0,+\infty)$. Let's consider the second case.

$$
\lim _{x \rightarrow 0+} f(x)=\lim _{x \rightarrow 0+}\left(x^{3}-\frac{1}{x}-1\right)=-\infty,
$$

due to the fact that $x^{3}$ and -1 are continuous in 0 .

$$
\lim _{x \rightarrow+\infty} f(x)=\lim _{x \rightarrow+\infty}\left(x^{3}-\frac{1}{x}-1\right)=+\infty
$$

due to the fact that $-\frac{1}{x} \rightarrow 0(x \rightarrow \infty)$. Because of the fact that

$$
f(x)=x^{3}-\frac{1}{x}-1
$$

is continuous on $(0,+\infty)$, it reaches every value from lower to upper bound, therefore, it reaches every value in $(-\infty,+\infty)$

