Answer on Question #44640 – Math - Trigonometry

Problem.

• How can a trigonometric function be chosen to model periodic phenomena with specified amplitude, frequency, and midline?

Solution.

Suppose that we have one dimensional case (object is moving only in direction) and the force F that are acting at the object depends directly upon the distance x (the assumptions are natural for many periodic physical systems: oscillation of the pendulum, vibrations of the string, oscillation of the spring etc.). Then by Newton's Second Law and the definition of acceleration

$$kx + b = F = ma = mx'',$$

where m is weight of object.

So we obtain a differential equation

$$x'' = \frac{k}{m}x + \frac{k}{b}.$$

The function

$$x = C_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}}t\right) - \frac{k}{b}$$

is the general solution of this equation. That is why functions $\cos(t)$ and $\sin(t)$ are often chosen to model periodic phenomena with specified amplitude, frequency, and midline. In any system both function can be used, as

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$

for all real α.

If we consider equation

$$x = A\cos(kt) + B$$

which is modeling some periodic phenomena, then A is its amplitude, B its midline and $\frac{2\pi}{k}$ is its frequency.