Answer on Question #44630 - Math - Other

Let $f(x) = e^x$ be defined on $[-1;1]$. For each positive integer $n = 3$, let P_n be the Lagrange interpolation polynomial of f at the n equidistant points in $[-1;1]$. Given a positive error δ, write a function that compute the smallest value of n so that $\|f - P_n\|_{L^\infty} < \delta$. For the test case, you can take $\delta = 10^{-4}$.

As $\|\cdot\|_{L^\infty(M)} = \mathrm{esssup}_{x\in M} |\cdot|$ and usually during programming we don't consider functions as *equivalence classes*, we can assume that $\|\cdot\|_{L^\infty(M)} = \max_{x\in M} |\cdot|$ Thus, below is written a script which implements required function

Listing 1 (from MatLab): Q44630.m

```
clc
clear all
%enter error
delta=0.0001
n=3;

syms x
f=exp(x);
err=1;
while err>delta
    X=-1:1/(n-2):1;
    Y=exp(X);

    %Lagrange

N=length(X);
W=1;
      for j=1:N
      W=W*(x-X(j));
      end;
W1=diff(W,x,1);
L=0;
      for i=1:N
      T=subs(W1,x,X(i));
      L=L+W.*Y(i)./(T.*(x-X(i)));
      end;
Lnew=simplify(L);
Lnew;
 W=exp(x)-Lnew;
 W=-abs(W);
      g=matlabFunction(W);
      m=fminbnd(g,-1,1);
err=double(subs(abs(W),x,m))

n=n+1;
end;
err
n-1
```