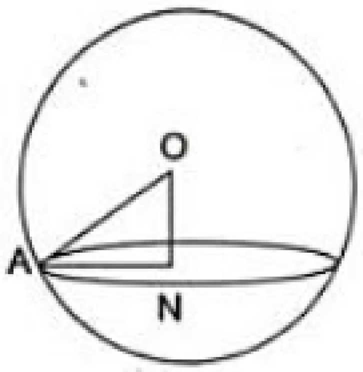
Answer on Question #44601 – Math - Analytic Geometry

Find the radius of the circular section of the sphere $x^2 + y^2 + z^2 = 49$ by the plane $2x + 3y - z - 5\sqrt{14} = 0$.

Solution:



The centre of the sphere is $O(0,0,0)$ and its radius = $OA = \sqrt{49} = 7$ here.

Now $ON =$ perpendicular distance from the centre of the sphere $O(0,0,0)$ to the plane $2x + 3y - z - 5\sqrt{14} = 0$. It is fairly clear from linear algebra that:

$$\mathrm{ON} = \frac{5\sqrt{14}}{\sqrt{2^2 + 3^2 + (-1)^2}} = 5$$

Then, since we know that the intersection of a plane and a sphere is always a circle, and that this distance is perpendicular to the circle, to find the radius of the circle, we reduce to solving the following:

Radius of circular section from the right triangle applying Pythagoras theorem:

$$ON^2 + AN^2 = AO^2$$

$$5^2 + \mathrm{r}^2 = 7^2\text{, or equivalently:}$$

$$\mathrm{r} = \sqrt{24}$$

Answer: $\mathrm{r} = \sqrt{24}$