Answer on Question #44599 – Math - Analytic Geometry

Problem.

Find the image of the point (1, 6,3) in the line
1 2
1 2 3
x y – z–
= =. Also find the equation of the line
joining the given point and its image.

Remark.

The statement isn't correctly formatted. I suppose that the correct statement is
"Find the image of the point (1, 6, 3) in the line

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}.$$

Also find the equation of the line joining the given point and its image."

Solution.

Let $A(1,6,3)$ and $l\colon \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Suppose that the perpendicular from A to the line l intersects the line l at B and image of point A is C. The equation of the line l can be rewritten, as

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = t$$

or $x = t, y = 2t - 1, z = 3t - 2$ where $t \in \mathbb{R}$.
Therefore point B has coordinates $(k, 2k + 1, 3k + 2)$, where k is unknown parameter.
The vector $\overrightarrow{AB}$ has coordinates $(k - 1, 2k + 1 - 6, 3k + 2 - 3) = (k - 1, 2k - 5, 3k - 1)$.
The direction vector of the line l has coordinates $(1, 2, 3)$.
The line l and the line $\overrightarrow{AB}$ are perpendicular, so the inner product of their direction vectors equals 0. Hence

$$(k-1)\cdot 1 + (2k-5)\cdot 2 + (3k-1)\cdot 3 = 0$$

or $k = 1$. Therefore $B(1,3,5)$.
The point C is the midpoint of the segment AB. If C has coordinates (x, y, z), then $\frac{x+1}{2} = 1, \frac{y+6}{2} = 3, \frac{z+3}{2} = 5$ or $x = 1, y = 0, z = 7$. $C(1,0,7)$.
The vector $\overrightarrow{AC}$ has coordinates $(1 - 1, 0 - 6, 7 - 3) = (0, -6, 4)$.
The equation of line AC is $x = 1, y = -6t + 6, z = 4t + 3$, where $t \in \mathbb{R}$.
Answer: $(1, 0, 7)$, $x = 1, y = -6t + 6, z = 4t + 3$, where $t \in \mathbb{R}$.