Answer on Question #44592 – Math – Other

A hospital records the number of floral deliveries its patients receive each day. For a two week period, the records show

15, 27, 26, 24, 18, 21, 26, 19, 15, 28, 25, 26, 17, 23

Use exponential smoothing with a smoothing constant of 0.4 to forecast the number of deliveries.

Solution

Unlike moving average models, which use a fixed number of the most recent values in the time series for smoothing and forecasting, exponential smoothing incorporates all values time series, placing the heaviest weight on the current data, and weights on older observations that diminish exponentially over time. Because of the emphasis on all previous periods in the data set, the exponential smoothing model is recursive. When a time series exhibits no strong or discernible seasonality or trend, the simplest form of exponential smoothing – single exponential smoothing – can be applied. The formula for single exponential smoothing is given by

$$\widehat{Y}_{t+1} = \alpha Y_t + (1-\alpha)\widehat{Y}_t$$

In this equation, $\widehat{Y}_{t+1}$ represents the forecast value for period t + 1; Y_t is the actual value of the current period, t; $\widehat{Y}_t$ is the forecast value for the current period, t; and α is the smoothing constant, or alpha, a number between 0 and 1. Alpha is the weight we assign to the most recent observation in our time series. Essentially, we are basing our forecast for the next period on the actual value for this period, and the value we forecasted for this period, which in turn was based on forecasts for periods before that.

We apply the noted above formula to our problem.

$$\widehat{Y}_2 = (1-\alpha)\widehat{Y}_1 + \alpha Y_1 = (1-0.4)\cdot 15 + 0.4\cdot 15 = 15$$

$$\widehat{Y}_3 = (1-\alpha)\widehat{Y}_2 + \alpha Y_2 = (1-0.4)\cdot 15 + 0.4\cdot 27 = 19.8$$

$$\widehat{Y}_4 = (1-\alpha)\widehat{Y}_3 + \alpha Y_3 = (1-0.4)\cdot 19.8 + 0.4\cdot 26 = 22.28$$

$$\widehat{Y}_5 = (1-\alpha)\widehat{Y}_4 + \alpha Y_4 = (1-0.4)\cdot 22.28 + 0.4\cdot 24 = 22.968$$

$$\widehat{Y}_6 = (1-\alpha)\widehat{Y}_5 + \alpha Y_5 = (1-0.4)\cdot 22.968 + 0.4\cdot 18 = 20.9808$$

$$\widehat{Y}_7 = (1-\alpha)\widehat{Y}_6 + \alpha Y_6 = (1-0.4)\cdot 20.9808 + 0.4\cdot 21 = 20.98848$$

$$\widehat{Y}_8 = (1-\alpha)\widehat{Y}_7 + \alpha Y_7 = (1-0.4)\cdot 20.98848 + 0.4\cdot 26 = 22.993088$$

$$\widehat{Y}_9 = (1-\alpha)\widehat{Y}_8 + \alpha Y_8 = (1-0.4)\cdot 22.993088 + 0.4\cdot 19 = 21.3958528$$

$$\widehat{Y}_{10} = (1-\alpha)\widehat{Y}_9 + \alpha Y_9 = (1-0.4)\cdot 21.3958528 + 0.4\cdot 15 = 18.83751168$$

$$\widehat{Y}_{11} = (1-\alpha)\widehat{Y}_{10} + \alpha Y_{10} = (1-0.4)\cdot 18.83751168 + 0.4\cdot 28 = 22.502507008$$

$$\widehat{Y}_{12} = (1-\alpha)\widehat{Y}_{11} + \alpha Y_{11} = (1-0.4)\cdot 22.502507008 + 0.4\cdot 25 = 23.5015042048$$

$$\widehat{Y}_{13} = (1-\alpha)\widehat{Y}_{12} + \alpha Y_{12} = (1-0.4)\cdot 23.5015042048 + 0.4\cdot 26 = 24.50090252288$$

$$\widehat{Y}_{14} = (1-\alpha)\widehat{Y}_{13} + \alpha Y_{13} = (1-0.4)\cdot 24.50090252288 + 0.4\cdot 17 = 21.500541513728$$

$$\widehat{Y}_{15} = (1-\alpha)\widehat{Y}_{13} + \alpha Y_{13} = (1-0.4)\cdot 21.500541513728 + 0.4\cdot 23 = 22.1003249082368$$

First, we provide the obtained results in Table 1.

Table 1 Actual and forecasting the number of deliveries with the smoothing constant $\alpha = 0.4$.

Time Period	**Time Series Value**	**Forecast**	**Forecast Error**
1	15	15	0
2	27	19.8	7.2
3	26	22.280	3.72
4	24	22.968	1.032
5	18	20.981	-2.981
6	21	20.988	0.012
7	26	22.993	3.007
8	19	21.396	-2.396
9	15	18.838	-3.838
10	28	22.503	5.497
11	25	23.502	1.498
12	26	24.501	1.499
13	17	21.501	-4.501
14	23	22.100	0.900

We can represent the results of calculations in Figure 1.

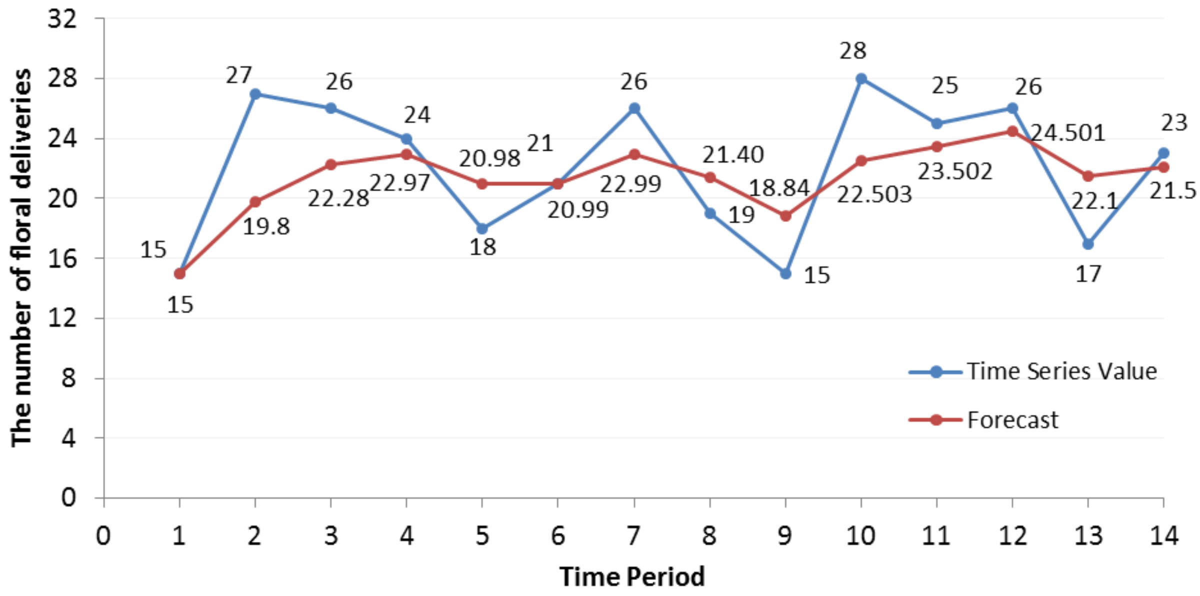

Figure 1 Actual and forecasting the number of deliveries with the smoothing constant $\alpha = 0.4$.