

Answer on Question #44584 – Math – Linear Algebra

**Question 1.** Check that the vectors  $u = (3/5, 4/5, 0)$ ,  $v = (-4/5, 3/5, 0)$  and  $w = (0, 0, 1)$  are orthonormal. Further, write the vector  $a = (1, -1, 2)$  as a linear combination of the vectors.

*Solution.* We have

$$u \cdot u = (3/5)^2 + (4/5)^2 + 0^2 = 9/25 + 16/25 + 0 = 25/25 = 1;$$

$$u \cdot v = (3/5)(-4/5) + (4/5)(3/5) + 0 \cdot 0 = -12/25 + 12/25 + 0 = 0;$$

$$u \cdot w = (3/5)0 + (4/5)0 + 0 \cdot 1 = 0 + 0 + 0 = 0;$$

$$v \cdot v = (-4/5)^2 + (3/5)^2 + 0^2 = 16/25 + 9/25 + 0 = 25/25 = 1;$$

$$v \cdot w = (-4/5)0 + (3/5)0 + 0 \cdot 1 = 0 + 0 + 0 = 0;$$

$$w \cdot w = 0^2 + 0^2 + 1^2 = 1,$$

so,  $u, v$  and  $w$  are orthonormal.

Let  $a = \alpha u + \beta v + \gamma w$ , that is

$$\begin{aligned}(1, -1, 2) &= \alpha(3/5, 4/5, 0) + \beta(-4/5, 3/5, 0) + \gamma(0, 0, 1) \\ &= ((3/5)\alpha - (4/5)\beta, (4/5)\alpha + (3/5)\beta, \gamma).\end{aligned}$$

This gives

$$(3/5)\alpha - (4/5)\beta = 1,$$

$$(4/5)\alpha + (3/5)\beta = -1,$$

$$\gamma = 2.$$

Multiplying the first equation by 20 and the second one by  $-15$ , we get

$$12\alpha - 16\beta = 20,$$

$$-12\alpha - 9\beta = 15.$$

Adding the equations, we obtain  $-25\beta = 35$ , so  $\beta = -7/5$ . Then

$$\alpha = (5/3)(1 + (4/5)\beta) = (5/3)(1 - 28/25) = (5/3)(-3/25) = -1/5.$$

Thus  $a = (-1/5)u + (-7/5)v + 2w$ . □