Question 1. Let a = $(1/(2\sqrt{2}), \sqrt{3}/(2\sqrt{2}), 1/\sqrt{2})$ and b = $(1/\sqrt{2}, 0, 1/\sqrt{2})$.

1. Find the direct cosines of a and b;
2. Find the angle between a and b.

Solution. (i) We have

$$|a| = \sqrt{\left(1/(2\sqrt{2})\right)^2 + \left(\sqrt{3}/2\sqrt{2}\right)^2 + \left(1/\sqrt{2}\right)^2}$$

$$= \sqrt{1/8 + 3/8 + 1/2}$$

$$= \sqrt{1}$$

$$= 1.$$

Hence, the direct cosines of a are

$\alpha_1 = \frac{1/(2\sqrt{2})}{1} = (1/(2\sqrt{2}));$

$\beta_1 = \frac{\sqrt{3}/(2\sqrt{2})}{1} = \sqrt{3}/(2\sqrt{2});$

$\gamma_1 = \frac{1/\sqrt{2}}{1} = 1/\sqrt{2}.$

Similarly

$$|b| = \sqrt{\left(1/\sqrt{2}\right)^2 + 0^2 + \left(1/\sqrt{2}\right)^2}$$

$$= \sqrt{1/2 + 0 + 1/2}$$

$$= \sqrt{1}$$

$$= 1.$$

Hence, the direct cosines of b are

$\alpha_2 = \frac{1/\sqrt{2}}{1} = 1/\sqrt{2};$

$\beta_2 = \frac{0}{1} = 0;$

$\gamma_2 = \frac{1/\sqrt{2}}{1} = 1/\sqrt{2}.$

(ii) Let θ denote the angle between a and b. Then

$$\cos \theta = \alpha_1 \alpha_2 + \beta_1 \beta_2 + \gamma_1 \gamma_2$$

$$= (1/(2\sqrt{2}))(1/\sqrt{2}) + (\sqrt{3}/(2\sqrt{2}))0 + (1/\sqrt{2})(1/\sqrt{2})$$

$$= 1/4 + 0 + 1/2$$

$$= 3/4.$$