

**Answer on Question #44583 - Math - Linear Algebra**

**Question 1.** Let  $a = (1/(2\sqrt{2}), \sqrt{3}/(2\sqrt{2}), 1/\sqrt{2})$  and  $b = (1/\sqrt{2}, 0, 1/\sqrt{2})$ .

1. Find the direct cosines of  $a$  and  $b$ ;
2. Find the angle between  $a$  and  $b$ .

**Solution.** (i) We have

$$\begin{aligned}
 |a| &= \sqrt{\left(1/(2\sqrt{2})\right)^2 + (\sqrt{3}/(2\sqrt{2}))^2 + (1/\sqrt{2})^2} \\
 &= \sqrt{1/8 + 3/8 + 1/2} \\
 &= \sqrt{1} \\
 &= 1.
 \end{aligned}$$

Hence, the direct cosines of  $a$  are

$$\begin{aligned}
 \alpha_1 &= \frac{1/(2\sqrt{2})}{1} = 1/(2\sqrt{2}); \\
 \beta_1 &= \frac{\sqrt{3}/(2\sqrt{2})}{1} = \sqrt{3}/(2\sqrt{2}); \\
 \gamma_1 &= \frac{1/\sqrt{2}}{1} = 1/\sqrt{2}.
 \end{aligned}$$

Similarly

$$\begin{aligned}
 |b| &= \sqrt{(1/\sqrt{2})^2 + 0^2 + (1/\sqrt{2})^2} \\
 &= \sqrt{1/2 + 0 + 1/2} \\
 &= \sqrt{1} \\
 &= 1.
 \end{aligned}$$

Hence, the direct cosines of  $b$  are

$$\begin{aligned}
 \alpha_2 &= \frac{1/\sqrt{2}}{1} = 1/\sqrt{2}; \\
 \beta_2 &= \frac{0}{1} = 0; \\
 \gamma_2 &= \frac{1/\sqrt{2}}{1} = 1/\sqrt{2}.
 \end{aligned}$$

(ii) Let  $\theta$  denote the angle between  $a$  and  $b$ . Then

$$\begin{aligned}
 \cos \theta &= \alpha_1 \alpha_2 + \beta_1 \beta_2 + \gamma_1 \gamma_2 \\
 &= (1/(2\sqrt{2}))(1/\sqrt{2}) + (\sqrt{3}/(2\sqrt{2}))0 + (1/\sqrt{2})(1/\sqrt{2}) \\
 &= 1/4 + 0 + 1/2 \\
 &= 3/4.
 \end{aligned}$$