Answer on Question #44581 – Math – Calculus

Question 1. Which of the following functions are 1 - 1 and which are onto? Justify your answer.

1. $f : \mathbb{R} \to \mathbb{R}_{\geq 0}$ given by $f(x) = x^2$ where $\mathbb{R}_{\geq 0}$ is the set $\{x \in \mathbb{R} |\ x \geq 0\}$;
2. $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2 + x + 1$.

Solution. (i) We have $f(-1) = f(1) = 1$, so f is not 1 - 1. For any $y \in \mathbb{R}_{\geq 0}$, since $y \geq 0$, there exists $x = \sqrt{y}$ such that $f(x) = (\sqrt{y})^2 = y$. Thus, f is onto.

(ii) We have $f(0) = f(-1) = 1$, so f is not 1 - 1. Observe that

$$f(x) = x^2 + x + 1 = x^2 + 2(1/2)x + 1/4 + 3/4 = (x + 1/2)^2 + 3/4 \geq 3/4.$$

In particular, there is no $x \in \mathbb{R}$, such that $f(x) = 0$. Thus, f is not onto.

Answer:

1. f is not 1 - 1, but onto;
2. f is neither 1 - 1, nor onto.